

Bound-state effects in $\mu^+e^- \rightarrow \gamma\gamma$ and $\bar{B}_s^0 \rightarrow \gamma\gamma$ decays

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Abstract. We demonstrate that in the double-radiative decays of heavy–light QED and QCD atoms, $\mu^+e^- \rightarrow \gamma\gamma$ and $\bar{B}_s^0 \rightarrow \gamma\gamma$, there is a contribution coming from operators that vanish on the free-quark mass shell. This off-shell effect is suppressed with respect to the effect of the well-known flavour-changing magnetic-moment operator by the bound-state binding factor. Accordingly, the negligible off-shellness of the weakly bound QED atoms becomes important for strongly bound QCD atoms. We present this effect in two different model approaches to QCD, one of them enabling us to keep close contact to the related effect in QED.

1 Introduction

In this paper we focus on the particular off-shell (or the binding) effects in the heavy–light fermion systems, common to QED and QCD. Such a comparative study throws light on the off-shell non-perturbative effects of valence quarks, studied first by two of us for the double-radiative decays of the K_L [1,2] and B_s meson [3]. Subsequently, this study has been continued within the specific bound-state models, both for $K_L \rightarrow 2\gamma$ [4] and for $\bar{B}_s^0 \rightarrow 2\gamma$ [5]. In these papers it was explicitly demonstrated that operators that vanish by using the perturbative equations of motion gave non-zero contributions for processes involving bound quarks. One of the purposes of the present paper is to demonstrate similar effects for the bound leptons.

To be specific, we consider such off-shell effects for two-photon annihilation of the μ^+e^- atom, called *muonium*. The off-shell effects will be given in terms of the binding factor characterising a given bound state. The role of this binding factor becomes more transparent in the case of the radiative decay of such a QED atom, where one deals with the simple Coulomb binding. This enables us to clearly demonstrate the off-shell effect in the QED case.

A careful study of these effects is motivated by the suitability of both lepton-changing transition $\mu \rightarrow e\gamma\gamma$, and $\bar{B}_s^0 \rightarrow \gamma\gamma$ decay, to test the standard model (SM) and to infer on the physics beyond the standard model (BSM).

By selecting the heavy–light muonium system μ^+e^- (where $m_\mu \equiv M \gg m_e \equiv m$), the bound-state calculation corresponds to that of the relativistic hydrogen one. Thereby we distinguish between the Coulomb field respon-

sible for the binding, and the radiation field [6] participating in the flavour-changing transition at the pertinent high-energy scale. In this way the radiative disintegration of an atom becomes tractable by implementing the two-step treatment [7]: “neglecting at first annihilation to compute the binding and then neglecting binding to compute annihilation”. For the muonium atom at hand, the binding problem is analogous to a solved problem of the H-atom. In this way we avoid the relativistic bound-state problem, which is a difficult subject, and we have no intention to contribute to it here.

This two-step method is known to work well for disintegration (annihilation) of the simplest QED atom, positronium. Generalisation of this procedure to muonium means that the two-photon decay width of muonium is obtained by using

$$\Gamma = \frac{|\psi(0)|^2 |\mathcal{M}(\mu^+e^- \rightarrow \gamma\gamma)|^2}{64\pi M m}, \quad (1)$$

where $|\psi(0)|^2$ is the square of the bound-state wave function at the origin. After this factorisation has been performed the rest of the problem reduces to the evaluation of the scattering–annihilation invariant amplitude \mathcal{M} . In the case of positronium this expression will involve equal masses ($M = m$), and the invariant amplitude which for the positronium annihilation at rest has a textbook form [8]

$$\mathcal{M} = \frac{ie^2}{2m^2} \bar{v}_s(p_2) \{ \not{\epsilon}_2^* \not{\epsilon}_1 \not{k}_1 + \not{\epsilon}_1^* \not{\epsilon}_2 \not{k}_2 \} u_r(p_1). \quad (2)$$

Only the antisymmetric piece in the decomposition of the product of three gamma matrices above,

$$\{ \} \rightarrow i\epsilon^{\mu\nu\alpha\beta} \gamma_5 \gamma_\beta (k_1 - k_2)_\alpha (\epsilon_1^*)_\mu (\epsilon_2^*)_\nu, \quad (3)$$

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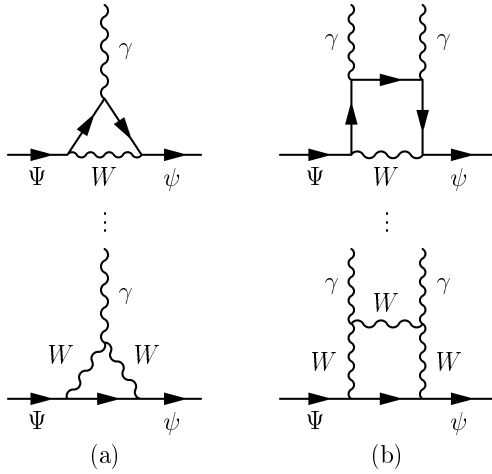


Fig. 1a,b. The examples of the one-particle-irreducible diagrams leading to the double-radiative flavour-changing transitions. Only the second-row diagrams exist for the leptonic case

contributes to the spin singlet parapositronium two-photon annihilation. This selects $(\epsilon_1^* \times \epsilon_2^*)$, a CP -odd configuration of the final two-photon state.

If parapositronium decay can serve as an initial benchmark in considering QED atom annihilation, then its QCD counterpart would be $\pi^0 \rightarrow \gamma\gamma$. However, the latter process shows some subtlety, known as the triangle anomaly. Interestingly enough, this quark atom double-radiative decay can also be viewed as an off-shell effect, as explained in some detail in [9]. It is the off-shellness in two-photon annihilation of atoms which we further explore in what follows.

This paper is organised as follows: In Sect. 2 we consider the quantum field treatment of the annihilation process $\mu^+e^- \rightarrow \gamma\gamma$ in arbitrary external field(s). In Sect. 3 we relate the binding forces to the external fields of Sect. 2. In Sect. 4 we consider the analogous heavy–light QCD system, and in Sect. 5 we give our conclusions.

2 Flavour-changing operators for $\mu^+e^- \rightarrow \gamma\gamma$

We treat the lepton flavour-changing process at hand analogously to the quark flavour change, accounted for by the electroweak theory. Thus, the double-radiative transition is triggered by two classes of one-particle-irreducible diagrams (Figs. 1a,b), related by the Ward identities. After integrating out the heavy particles in the loops, these one-loop electroweak transitions can be combined into an effective Lagrangian [1],

$$\mathcal{L}(e \rightarrow \mu)_\gamma = B\epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} (\bar{\Psi} i \overleftrightarrow{D}_\lambda \gamma_\rho L \psi) + \text{h.c.}, \quad (4)$$

where the muon and the electron are described by quantum fields $\Psi = \psi_\mu$ and $\psi = \psi_e$. Correspondingly, for $\bar{B}_s^0 \rightarrow 2\gamma$, the involved fields are $\psi_s = s$ and $\psi_b = b$.

In our case, we do not need to specify the physics behind the lepton-flavour-violating transition in (4). For in-

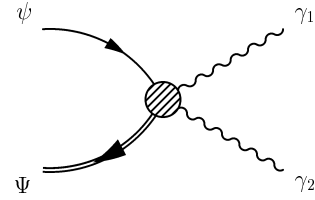


Fig. 2. The two-photon contact (seagull) diagram that can be rotated away by a field redefinition

stance, the strength B might contain some leptonic parameters, analogous to the Cabibbo–Kobayashi–Maskawa parameters λ_{CKM} in the quark sector.

Keeping in mind that the fermions in the bound states are not on-shell, we are not simplifying the result of the electroweak loop calculation by using the perturbative equation of motion. Thus the effective Lagrangian (4) obtained within perturbation theory splits into the on-shell magnetic transition operator \mathcal{L}_σ

$$\mathcal{L}_\sigma(1\gamma) = B_\sigma \bar{\Psi} (M \sigma \cdot FL + m \sigma \cdot FR) \psi + \text{h.c.}, \quad (5)$$

and an off-shell piece \mathcal{L}_F [1]

$$\mathcal{L}_F = B_F \bar{\Psi} [(i \overleftarrow{\not{D}} - M) \sigma \cdot FL + \sigma \cdot FR (i \not{D} - m)] \psi + \text{h.c.}, \quad (6)$$

where $\sigma \cdot F$ denotes $\sigma_{\mu\nu} F^{\mu\nu}$, and $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$ denote left-hand and right-hand projectors. To lowest order in QED (or QCD) $B_F = B_\sigma = B$, but in general they are different due to different anomalous dimensions of the operators in (5) and (6). (The off-shell part \mathcal{L}_F has zero anomalous dimension.)

By decomposing the covariant derivative $i \not{D} = i \not{\partial} - e \not{A}$ in the off-shell operator (6), we separate the one-photon piece

$$\mathcal{L}_F(1\gamma) = B_F \bar{\Psi} [(i \overleftarrow{\not{\partial}} - M) \sigma \cdot FL + \sigma \cdot FR (i \not{\partial} - m)] \psi + \text{h.c.}, \quad (7)$$

from the two-photon piece

$$\mathcal{L}_F(2\gamma) = B_F \bar{\Psi} [-e \not{A} \sigma \cdot FL + \sigma \cdot FR (-e \not{A})] \psi + \text{h.c.} \quad (8)$$

The amplitude for the two-photon diagram (Fig. 2) is given by

$$A_a = i \int d^4x \mathcal{L}_F(2\gamma) = A_a^L + A_a^R, \quad (9)$$

in an obvious notation. The single-photon off-shell Lagrangian $\mathcal{L}_F(1\gamma)$ leads to the amplitude with the heavy particle in the propagator

$$A_b = i B_F \int \int d^4x d^4y \bar{\Psi}(y) [-ie \not{A}_2(y)] i S_F^{(\mu)}(y-x) \times \left[(i \overleftarrow{\not{\partial}}_x - M) \sigma \cdot F_1(x) L + \sigma \cdot F_1(x) R (i \not{\partial}_x - m) \right] \psi(x), \quad (10)$$

and a similar amplitude with the light particle in the propagator

$$\begin{aligned}
A_c &= iB_F \int \int d^4x d^4y \\
&\times \bar{\Psi}(x) \left[(i\overleftarrow{\not{\partial}}_x - M)\sigma \cdot F_1(x)L + \sigma \cdot F_1(x)R(i\overrightarrow{\not{\partial}}_x - m) \right] \\
&\times iS_F^{(e)}(x-y) [-ie\mathcal{A}_2(y)] \psi(y).
\end{aligned} \tag{11}$$

The subscripts 1 and 2 distinguish between the two photons. It is understood that a term with the $1 \leftrightarrow 2$ subscript interchange should be added in order to make our result symmetric in the two photons.

Within the quantum field formalism, the sum of the (9), (10) and (11) describes the process $\mu^+e^- \rightarrow \gamma\gamma$, or $\mu \rightarrow e\gamma\gamma$.

Let us now be very general, and assume that both particles (e and μ) feel some kind of external field(s) represented by $V_{(e)}$ and $V_{(\mu)}$, and obey one-body Dirac equations

$$[i\overrightarrow{\not{\partial}} - V_{(i)}(x) - m_{(i)}] \psi_{(i)} = 0, \tag{12}$$

for $i = e$ or μ (in general $V_{(i)} = \gamma_\alpha V_{(i)}^\alpha$), and accordingly the particle propagators $S_F^{(i)}$ satisfy

$$[i\overrightarrow{\not{\partial}} - V_{(i)}(x) - m_i] S_F^{(i)}(x-y) = \delta^{(4)}(x-y). \tag{13}$$

Our photon fields enter via perturbative QED, switched on by the replacement $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$ in (12). It should be emphasised that $A_\mu(x)$ represents the radiation field and does not include binding forces, which will in the next section be related to the external fields $V_{(i)}$.

Now, using relations (12) and (13) we obtain

$$A_b = -A_a^L + \Delta A_b, \quad A_c = -A_a^R + \Delta A_c, \tag{14}$$

resulting in a partial cancellation when the amplitudes are summed

$$A_a + A_b + A_c = \Delta A_b + \Delta A_c. \tag{15}$$

This shows that the local off-diagonal fermion seagull transition of Fig. 2 cancels, even if the external fermions are off-shell. The quantities left over, ΔA_b and ΔA_c , involve integrals over the Coulomb potential and represent the net off-shell effect.

There are also amplitudes A_d and A_e which are counterparts of A_b and A_c when $\mathcal{L}_F(1\gamma)$ is replaced by \mathcal{L}_σ . The total contribution from our flavour-changing Lagrangian (\mathcal{L}_F and \mathcal{L}_σ parts) is then given by

$$\begin{aligned}
A_d + \Delta A_b &= i \int \int d^4x d^4y \bar{\Psi}(y) [-ie\mathcal{A}_2(y)] \\
&\times iS_F^{(\mu)}(y-x)Q(x)\psi(x),
\end{aligned} \tag{16}$$

represented by Fig. 3a, and a similar one,

$$\begin{aligned}
A_e + \Delta A_c &= i \int \int d^4x d^4y \bar{\Psi}(x)Q(x)iS_F^{(e)}(x-y) \\
&\times [-ie\mathcal{A}_2(y)] \psi(y),
\end{aligned} \tag{17}$$

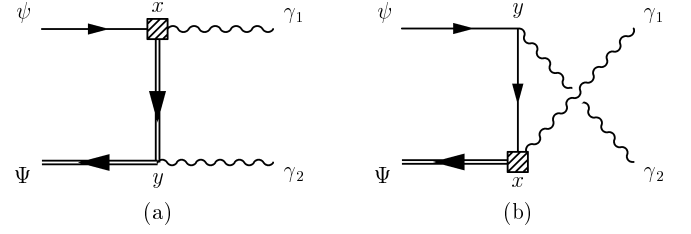


Fig. 3a,b. The shaded boxes indicate the combination of the unrotated off-shell transition ($\sim B_F$) and the on-shell magnetic-moment transition ($\sim B_\sigma$), giving the effective vertex in (26)

corresponding to Fig. 3b. The operator $Q(x)$ in these expressions reads

$$\begin{aligned}
Q(x) &= [B_\sigma M + B_F V_{(\mu)}(x)] \sigma \cdot F_1(x)L \\
&+ \sigma \cdot F_1(x)R [B_\sigma m + B_F V_{(e)}(x)].
\end{aligned} \tag{18}$$

The result given by (16)–(18) can also be understood in terms of the following field redefinition. Equation (12) can be obtained from the Lagrangian

$$\mathcal{L}_D(\Psi, \psi) = \bar{\Psi} [i\overrightarrow{\not{\partial}} - V_{(\mu)} - M] \Psi + \bar{\psi} [i\overrightarrow{\not{\partial}} - V_{(e)} - m] \psi. \tag{19}$$

Now, by defining new fields

$$\Psi' = \Psi + B_F \sigma \cdot FL \psi, \quad \psi' = \psi + B_F^* \sigma \cdot FL \Psi, \tag{20}$$

we obtain

$$\mathcal{L}_D(\psi, \Psi) + \mathcal{L}_F = \mathcal{L}_D(\psi', \Psi') + \Delta \mathcal{L}_B, \tag{21}$$

which shows that \mathcal{L}_F can be transformed away from the perturbative terms, but a relic of it,

$$\Delta \mathcal{L}_B = B_F \bar{\Psi} [V_{(\mu)} \sigma \cdot FL + \sigma \cdot F R V_{(e)}] \psi + \text{h.c.}, \tag{22}$$

remains in the bound-state dynamics. Thus, the off-shell effects are non-zero for bound external fermions. Combining $\Delta \mathcal{L}_B$ and \mathcal{L}_σ , we obtain

$$\Delta \mathcal{L}_B + \mathcal{L}_\sigma = \bar{\Psi} Q \psi + \text{h.c.}, \tag{23}$$

where Q is given by (18). This shows the equivalence of this field redefinition procedure and the result given by (16)–(18).

This is how far we can push the problem within quantum field theory. Up to now we have made no approximations except for standard perturbation theory. In the next section we will adapt the results of this section to the relevant bound-state effects.

3 Off-shellness in the muonium annihilation amplitude

As announced in the Introduction, we choose the simplest heavy–light QED atom, muonium. Naively, the product $\bar{\Psi}\psi$ corresponds to the bound state of μ^+ and e^- (which

might be true only in the asymptotically free case). However, relativistic bound-state physics is a difficult subject, out of our scope. We will stick to the two-step procedure [7] as explained in the Introduction. We perform the calculations in the muonium rest frame (CM frame of μ^+ and e^-) where we put the external field(s) equal to a mutual Coulomb field, $V_{(i)} \rightarrow \gamma_0 V_C$ (where $V_C = -e^2/4\pi r$). In calculating the $\mu^+e^- \rightarrow \gamma\gamma$ amplitude in momentum space, we take for V_C the average over solutions in the Coulomb potential, which is $\langle V_C \rangle = -(m\alpha^2/2)$. In this way the muonium-decay invariant amplitude acquires the form which is a straightforward generalisation of the positronium-decay invariant amplitude (2) in momentum space.

The amplitudes $A_d + \Delta A_b$ from (16), together with $A_e + \Delta A_c$ from (17), transformed to the momentum space take the form

$$\mathcal{M} = \frac{2eB_\sigma}{m} \bar{v}_\mu(p_2) \left\{ \frac{m}{M} \not{k}_2 \not{\epsilon}_2^* P - P \not{\epsilon}_2^* \not{k}_2 + (1 \leftrightarrow 2) \right\} u_e(p_1), \quad (24)$$

where v_μ and u_e are muon and electron spinors, and $\epsilon_{1,2}^*$ are photon polarisation vectors. The factor incorporating the binding in the form of a four-vector $U^\alpha = (\rho, \mathbf{0})$,

$$P \equiv (1 - x\not{U}) \not{k}_1 \not{\epsilon}_1^* L + x \not{k}_1 \not{\epsilon}_1^* R (1 - \not{U}), \quad (25)$$

accounts for the aforementioned factorisation of a binding and a decay, and is represented by the shaded box of Fig. 3:

$$\left[M(1 - x\rho\gamma^0)\sigma \cdot F_1 L + m\sigma \cdot F_1 R(1 - \rho\gamma^0) \right]. \quad (26)$$

Here we introduced abbreviations for two small constant parameters,

$$x \equiv \frac{m}{M}, \quad \rho \equiv -\frac{B_F \langle V_C \rangle}{mB_\sigma}, \quad (27)$$

in terms of which the sought off-shell effect will be expressed. Note that in the effective interaction (26), the left-handed part corresponding to $V_{(\mu)}$ has gotten an extra suppression factor $x = m/M$ in front of the binding factor ρ , in agreement with the expectation that the heavy particle (μ^+) is approximately free, and the light particle (e^-) is approximately the reduced particle, in analogy with the H-atom.

The annihilation amplitude (24) can now be evaluated explicitly. A tedious calculation, performed in the muonium rest frame with photons emitted along the z -axis, gives

$$\mathcal{M} = -2eB_\sigma M^2 \sqrt{\frac{2M}{m}} \left[(1 + x\rho) \epsilon_2^* \cdot \epsilon_1^* + i(1 + 2x + x\rho) (\epsilon_2^* \times \epsilon_1^*) \cdot \hat{k}_1 + \mathcal{O}(\rho^2, x^2) \right], \quad (28)$$

where we have kept only the leading terms in ρ and x . In comparison to the expressions (2) and (3) for parapositronium, we notice that in addition to $\epsilon_2^* \times \epsilon_1^*$ there appears also $\epsilon_2^* \cdot \epsilon_1^*$, a CP -even two-photon configuration.

The explicit expression for ρ depends on some assumptions. As explained previously, we use $\langle V_C \rangle = -m\alpha^2/2$ which gives $\rho = \alpha^2/2$ for $B_\sigma = B_F = B$, which is a good approximation in the leptonic case.

Equation (1) finally gives

$$\Gamma = \frac{2\alpha M^4}{m^2} |\psi(0)|^2 |B_\sigma|^2 (1 + 2x\rho). \quad (29)$$

Thus, for muonium, the sought off-shell contribution is only a tiny correction, $2x\rho = \alpha^2 m/M \simeq 2.6 \cdot 10^{-7}$, to the magnetic moment dominated rate¹. However, the corresponding off-shellness in a strongly bound QCD system should be significantly larger. We also take into account the B_F/B_σ correction in (29), when considering the $\bar{B}_s^0 \rightarrow \gamma\gamma$ decay below.

Before ending this section, we should also mention that the Lagrangian given by (18) and (23) can be used to calculate the amplitude for muonic hydrogen decaying to a photon and ordinary hydrogen, that is, the process $\mu^- \rightarrow e^- + \gamma$ for both leptons bound to a proton. This is a leptonic version of the celebrated B -meson decay $B_d \rightarrow K^* \gamma$.

As a toy model, one might consider a process “ μ ” \rightarrow “ e ” γ in an external Coulomb field, with “ μ ” and “ e ” rather close in mass such that the non-relativistic descriptions of the “leptons” might be used. The effective “ μ ” \rightarrow “ e ” γ interaction is given in (23). If we assume that $(M - m)$ is of order αm , we obtain off-shell effects of order α^2 due to \mathcal{L}_F , relative to the standard magnetic-moment term \mathcal{L}_σ . Bigger mass differences gives bigger effects, until the non-relativistic approximation breaks down.

4 Off-shellness in $\bar{B}_s^0 \rightarrow \gamma\gamma$

By the replacements $\mu \rightarrow s$ and $e \rightarrow b$, the expressions (4) to (8) apply to $b \rightarrow s\gamma\gamma$ induced $\bar{B}_s^0 \rightarrow 2\gamma$ decay amplitude. Then one has to scale the operators $\mathcal{L}_{F,\sigma}$ defined at the M_W scale, down to the B -meson scale. The coefficients B_F of \mathcal{L}_F , and B_σ of \mathcal{L}_σ , in (6) and (5), both being equal to B at the W scale, may evolve differently down to the $\mu = m_b$ scale. This difference between B_F and B_σ is due to different anomalous dimensions of the respective operators. Within the SM one can write

$$B_{\sigma,F} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \frac{e}{16\pi^2} C_7^{\sigma,F}. \quad (30)$$

The coefficient C_7^σ has been studied by various authors [10–14]. The coefficient C_7^F was considered in [3], where at the b -quark scale we obtained

$$\frac{C_7^F}{C_7^\sigma} \simeq 4/3 \quad (\mu = m_b). \quad (31)$$

Although the off-shell effect for $B \rightarrow 2\gamma$ is expected to be suppressed by the ratio (binding energy)/ m_b , it could still be numerically interesting.

¹ Note that it is not necessary to know the precise value of $|\psi(0)|^2 \sim (m\alpha)^3/\pi$, in order to know the relative off-shell contribution

4.1 Coulomb-type QCD model

The conventional procedure when evaluating the pseudoscalar meson decay amplitudes is to express them in terms of the meson decay constants, by using the PCAC relations

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}_s^0(P) \rangle = -i f_B P_\mu, \quad (32)$$

$$\langle 0 | \bar{s} \gamma_5 b | \bar{B}_s^0(P) \rangle = i f_B M_B. \quad (33)$$

These relations will be useful after reducing our general expression (24) containing the terms with products of up to five gamma matrices. After some calculation we arrive at the expression for the B_s -meson decay at rest, which is analogous to, and in fact confirms our previous relation (28) obtained in different way,

$$\begin{aligned} \mathcal{M}^B = & -i \frac{e}{3} B_\sigma f_B M^2 \frac{(1+x)^2}{x} [(1+x\tau)\epsilon_2^* \cdot \epsilon_1^* \\ & + i(1+2x+x\tau)(\epsilon_2^* \times \epsilon_1^*) \cdot \hat{\mathbf{k}}_1 + \mathcal{O}(\tau^2, x^2)]. \end{aligned} \quad (34)$$

Here, the parameter τ represents the off-shell effect in the QCD problem at hand, and will be more model dependent than its QED counterpart ρ . With the amplitude (34) we arrive at the total decay width

$$\Gamma = \frac{\alpha M^5}{18m^2} f_B^2 |B_\sigma|^2 (1+2x\tau), \quad (35)$$

where by switching off τ we reproduce the result of [15].

In order to estimate the value of the off-shell contribution τ , in this subsection we assume a QED-like QCD model with the Coulombic wave function [16, 17] $\psi(r) \sim \exp(-mr\alpha_{\text{eff}})$. Thus we rely again on an exact solution corresponding to effective potential $V(r) = -4\alpha_{\text{eff}}/(3r)$, with effective coupling $\alpha_{\text{eff}}(r) = -(4\pi b_0 \ln(r\Lambda_{\text{pot}}))^{-1}$. Here $b_0 = (1/8\pi^2)(11 - (2/3)N_f)$. The mass scale Λ_{pot} appropriate to the heavy-light quark $\bar{Q}q$ potential is related to the more familiar QCD scale parameter, e.g. $\Lambda_{\text{pot}} = 2.23\Lambda_{\overline{\text{MS}}}$ (for $N_f=3$). Within this model, we obtain

$$\tau = \frac{2}{3} \alpha_{\text{eff}}^2 \frac{C_7^F}{C_7^\sigma}. \quad (36)$$

By matching the meson decay constant f_B and the wave function at the origin

$$N_c \frac{|\psi_B(0)|^2}{M} = \left(\frac{f_B}{2}\right)^2, \quad |\psi_B(0)|^2 = \frac{(m\alpha_{\text{eff}})^3}{\pi}, \quad (37)$$

we obtain the value for the strong interaction fine structure strength $\alpha_{\text{eff}} \sim 1$. Then, including (31) for the QCD case, the correction factor

$$x\tau \simeq 0.1 \quad (38)$$

is much larger than $x\rho$ in the corresponding QED case. Correspondingly, one expects even more significant off-shell effects in light quark systems, in compliance with our previous results [1, 2, 4].

4.2 A constituent quark calculation

Now we adopt a variant of the approach in [3, 5] as an alternative to the Coulomb-type QCD model described above. One might use the PCAC relations (32) and (33) together with a kinematical assumption for the \bar{s} -quark momentum, similar to those in [15, 18]. Assuming the bound \bar{s} and b quarks in \bar{B}_s^0 to be on their respective *effective* mass shells (effective mass being current mass plus a constituent mass m_0 of order 200–300 MeV), the structure of the amplitude comes out essentially as in (34) with a relative off-shell contribution

$$x\tilde{\tau} = \frac{2m_0}{m_b} \sim 0.1, \quad (39)$$

of the same order as in (38). However, unlike (34), the off-shell effect is now only in the CP -odd term ($\epsilon_1^* \times \epsilon_2^*$), the square bracket in (34) being replaced by

$$[\epsilon_2^* \cdot \epsilon_1^* + i(1+2x+x\tilde{\tau})(\epsilon_2^* \times \epsilon_1^*)]. \quad (40)$$

This may be different in other approaches [2], showing the model dependence of the off-shell effect. For instance, potential-QCD models in general, besides a vector Coulomb potential, also contain a scalar potential.

4.3 A bound-state quark model

For $\bar{B}_s^0 \rightarrow 2\gamma$, we have previously [3, 5] applied a bound-state model, where the potentials V_i in (12) are replaced by a quark-meson interaction Lagrangian

$$\mathcal{L}_\Phi(s, b) = G_B \bar{b} \gamma_5 s \Phi + \text{h.c.}, \quad (41)$$

where Φ is the B -meson field. Then, the term \mathcal{L}_F can be transformed away by means of the field redefinitions:

$$s' = s + B_F \sigma \cdot FLb, \quad b' = b + B_F^* \sigma \cdot FLs. \quad (42)$$

However, its effect reappears in a new bound-state interaction $\Delta\mathcal{L}_\Phi$,

$$\mathcal{L}_\Phi(s, b) + \mathcal{L}_F = \mathcal{L}_\Phi(s', b') + \Delta\mathcal{L}_\Phi, \quad (43)$$

where, after using $R\gamma_5 = R$ and $L\gamma_5 = -L$,

$$\Delta\mathcal{L}_\Phi = B_F G_B [\bar{b}' \sigma \cdot FLb' - \bar{s}' \sigma \cdot FRs'] \Phi + \text{h.c.} \quad (44)$$

Also in these cases [3, 5], the net off-shell effects are found. Further calculations of $B \rightarrow 2\gamma$ within bound-state models of the type in (41) will be presented elsewhere.

5 Conclusions

We have demonstrated the appearance of the off-shell effects in the flavour-changing two-photon decay of muonium and its hadronic $\bar{B}_s^0 \rightarrow \gamma\gamma$ counterpart. It is a quite significant 10 percent effect in the latter case, whereas

in the leptonic case it is very small (of order 10^{-7}), but clearly identifiable.

The present “atomic” approach enables us to see in a new light the effect studied first for the $K_L \rightarrow \gamma\gamma$ amplitude in the chiral quark model [1,2], and subsequently in the bound-state model [4]. The observation that off-shell effects can be clearly isolated from the rest in the heavy–light quark atoms [3] was still plagued by the uncertainty in the QCD binding calculation [5]. Here, in the Coulomb-type QCD model we are able to subsume the effect into an universal binding factor, in the same way as for the two-photon decay of muonium in the exactly solvable QED framework. As a result, we obtain the explicit expressions describing how the flavour-changing operators that vanish on-shell modify both the CP -even ($\epsilon_1^* \cdot \epsilon_2^*$), and CP -odd ($\epsilon_1^* \times \epsilon_2^*$) configuration of the final photons. In a constituent quark calculation we get a similar result for the off-shell effect. As a difference, in this case the off-shellness resides solely in the CP -odd part of the amplitude.

The main result of the present paper is a clear demonstration of the parallelism of the strict non-zero off-shell effects in the leptonic and quark heavy–light systems. Thus, we have established a solid ground for estimating the off-shell bound-state effects in the important $B_d \rightarrow K^*\gamma$ decay, which will be presented elsewhere [19].

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